Ex.16　证明. 记矩阵$A = (\alpha_1, \alpha_2, \cdots, \alpha_m)$, $B = (\beta_1, \beta_2, \cdots, \beta_m)$, 则有$B = AK$, 其中,

$$K = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{pmatrix}.$$

考虑齐次线性方程组

$$x_1\beta_1 + x_2\beta_2 + \cdots + x_m\beta_m = 0.$$

即$Bx = 0$. 由$B = AK$ 得$AKx = 0$. 又向量组$\alpha_1, \alpha_2, \cdots, \alpha_r$ 线性无关, 因此, $Kx = 0$. 显然, 向量组$\beta_1, \beta_2, \cdots, \beta_m$ 线性无关的充要条件是齐次线性方程组$Bx = 0$ 只有零解, 这等价于$|K| \neq 0$.